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Relationship of Unsteadiness in Downwash to the Quality of Parameter Estimates

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This paper investigates the relative importance of including unsteady effects in the lift and downwash in the longitudinal dynamics and parameter extraction algorithm. A simple vortex system has been used to model unsteady incompressible aerodynamic effects into the longitudinal equations of motion of an aircraft. Computer-generated data and flight data were used to demonstrate that inclusion of unsteady aerodynamics in the parameter-extraction algorithm produced aerodynamic parameters that were different from those extracted when unsteady aerodynamics were left out of the algorithm. The differences between derivatives associated with the two extraction algorithms (with and without unsteady aerodynamics) were related to acceleration derivatives which usually cannot be extracted individually.

Nomenclature

c	= chord
C_L	= lift coefficient, lift$/\bar{q}S_w$
$\Delta C_L(t)$	= lift due to unit step in α
C_m	= pitching moment coefficient, pitching moment$/\bar{q}S_w c_w$
$Ei(\ )$	= exponential integral of variable ( )
j	$=\sqrt{-1}$
I_y	= moment of inertia about y axis
ℓ	= distance behind wing root quarter-chord point
m	= aircraft mass
q	= pitch rate
$\bar{q}$	= dynamic pressure
S	= area
t,τ	= time
U	= velocity
α	= angle of attack
δ_e	= control deflection
ϵ	= downwash angle
$\Delta\epsilon(t)$	= response in downwash to a unit step in α
ω	= angular frequency
ρ	= air density

Subscripts

ss	= steady state (no unsteady aerodynamic effects)
w	= wing
t	= horizontal tail
f	= fuselage

Superscript

$(\tilde{\ })$	= variable ( ) in the frequency domain

Derivatives

C_{L_α}	$=\dfrac{\partial C_L}{\partial \alpha}$
$C_{L\delta_e}$	$=\dfrac{\partial C_L}{\partial \delta_e}$
C_{L_q}	$=\dfrac{\partial C_L}{\partial (qc/2U)}$
C_{m_α}	$=\dfrac{\partial C_m}{\partial \alpha}$
$C_{m\delta_e}$	$=\dfrac{\partial C_m}{\partial \delta_e}$
C_{m_q}	$=\dfrac{\partial C_m}{\partial (qc/2U)}$
$C_{m_{\dot\alpha}}$	$=\dfrac{\partial C_m}{\partial (\dot\alpha c/2U)}$

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Introduction

DIFFERENCES in extracted parameters, particularly in their variances, generally have been attributed to insufficient excitation of the aircraft and strong correlations between some of the parameters. However, another possible reason is that unsteady aerodynamics have not been sufficiently modeled.

Although there are various methods available for calculating unsteady aerodynamic loads on lifting surfaces[1-3] the availability of a simplified but accurate method has appeal to many aircraft designers. References 4 and 5 developed a simplified method for including unsteady effects in the longitudinal equations of motion for use in parameter identification. In those references, a simple vortex system was developed for estimating indicial lift and downwash for unswept wings in incompressible flow. It was shown that it gave results in good agreement with more accurate and complex vortex systems.

One purpose of the present work is to further develop the unsteady incompressible aerodynamic model and estimation algorithm of Refs. 4 and 5 and apply the algorithm to flight data. In particular, attention is given to unsteady effects in the downwash (the phase lag or "time-lag" of the downwash) and to indicial lift buildup. Statler[6] investigated the effect of unsteadiness in downwash and concluded that this effect could be accounted for using quasisteady theory with the classical time-lag of downwash concept. This fact was further borne out in the present paper.

Analysis

Equations of Motion: Time Domain

The perturbed short period longitudinal equations of motion in terms of angle of attack and pitch rate are

$$\dot{\alpha}(t) = q(t) - (\rho U S_w)/(2m) C_L(t) \tag{1}$$

$$\dot{q}(t) = (\rho U^2 S_w c_w)/(2I_y) C_m(t) \tag{2}$$

The unsteady aerodynamic effects are modeled in the coefficients $C_L(t)$ and $C_m(t)$ according to

$$C_L(t) = C_{L_w}(t) + C_{L_t}(t) = C_{L\delta_e}\delta_e(t) \tag{3}$$

$$C_m(t) = \ell/c_w C_{L_t}(t) + C_{m\delta_e}\delta_e(t) + (C_{m_\alpha})_f \alpha(t) \tag{4}$$

$$C_{L_w}(t) = \int_0^t \Delta C_{L_w}(t-\tau)\dot{\alpha}(\tau)\mathrm{d}\tau \tag{5}$$

$$C_{L_t}(t) = \int_0^t \Delta C_{L_t}(t-\tau)[\dot{\alpha}(\tau) - \dot{\epsilon}(\tau) + \ell/U \dot{q}(\tau)]\mathrm{d}\tau \tag{6}$$

$$\epsilon(t) = \int_0^t \Delta\epsilon(t-\tau)\dot{\alpha}(\tau)\mathrm{d}\tau \tag{7}$$

Expressions for the indicial lift and downwash are given as[5]

$$\Delta C_L(t) = (C_{L\alpha})_{ss}[1 - y\exp(-2zUt/c)] \tag{8}$$

$$\Delta\epsilon(t) = \left(\frac{\partial\epsilon}{\partial\alpha}\right)_{ss}\{1 + F[1-(\ell - Ut)/c]^{-1} - G\exp(-2HUt/c)\} \tag{9}$$

where the constants $F,G,H,y,z,(C_{L\alpha})_{ss}$ and $(\partial\epsilon/\partial\alpha)_{ss}$ are expressed as functions of the wing geometry.

Modification of the longitudinal equations of motion to incorporate unsteady aerodynamics results in integro-differential equations. Numerical integration is time consuming and, for this reason, the estimation was performed in the frequency domain.

Equations of Motion: Frequency Domain

The Fourier transform pair associated with a function $x(t)$ is

$$\tilde{x}(j\omega) = \int_0^\infty x(t)e^{-j\omega t}\mathrm{d}t \tag{10}$$

$$x(t) = (\tfrac{1}{2}\pi)\int_{-\infty}^{\infty}\tilde{x}(j\omega)e^{j\omega t}\mathrm{d}\omega \tag{11}$$

Equations (1) and (2) transform into

$$j\omega\tilde{\alpha}(j\omega) = \tilde{q}(j\omega) - (\rho U S_w)/(2m)\{[N_w(j\omega) + \tilde{C}_{L_t}(j\omega)]\tilde{\alpha}(j\omega) + (\ell/U)N_t(j\omega)\tilde{q}(j\omega) + C_{L_{\delta_e}}\tilde{\delta}_e(j\omega)\} \tag{12}$$

$$j\omega\tilde{q}(j\omega) = (\rho U S_w c_w)/(2I_y)\{C_{m_{\delta_e}}\tilde{\delta}_e(j\omega) - (\ell^2)/(c_w U)N_t(j\omega)\tilde{q}(j\omega) - (\ell/c_w)C_{L_t}(j\omega)\tilde{\alpha}(j\omega) + (C_{m_\alpha})_f\tilde{\alpha}(j\omega)\} \tag{13}$$

where

$$\tilde{C}_{L_t}(j\omega) = N_t(j\omega)\left[1 - \left(\frac{\partial\epsilon}{\partial\alpha}\right)_{ss} + \tilde{D}(j\omega)\right] \tag{14}$$

and

$$N(j\omega) = (C_{L_\alpha})_{ss}[1 - j\omega y(j\omega + 2zU/c_w)^{-1}] \tag{15}$$

$$\tilde{D}(j\omega) = \left(\frac{\partial\epsilon}{\partial\alpha}\right)_{ss}\{(j\omega + 2HU/c_w)^{-1}j\omega G - j\omega c_w(F/U) \times Ei[(\ell - c_w)j\omega/U]\exp[(\ell - c_w)j\omega/U]\} \tag{16}$$

For the case in which indicial lift is neglected ($y = 0$), the equations of motion are written as

$$j\omega\tilde{\alpha}(j\omega) = \tilde{q}(j\omega) - (\rho U S_w)/(2m)\{[C_{L_\alpha} + (C_{L\alpha})_{ss,t} \times \tilde{D}(j\omega)]\tilde{\alpha}(j\omega) + (c_w)/(2U)C_{L_q}\tilde{q}(j\omega) + C_{L\delta_e}\tilde{\delta}_e(j\omega)\} \tag{17}$$

$$j\omega\tilde{q}(j\omega) = (\rho U^2 S_w c_w)/(2I_y)\{[(C_{m_\alpha}) - (\ell/c_w)(C_{L\alpha})_{ss,t} \times \tilde{D}(j\omega)]\tilde{\alpha}(j\omega) + (c_w)/(2U)C_{m_q}\tilde{q}(j\omega) + C_{m_{\delta_e}}\tilde{\delta}_e(j\omega)\} \tag{18}$$

where

$$C_{L_q} = (2\ell/c_w)(C_{L_\alpha})_{ss,t}$$

$$C_{m_q} = -2(\ell/c_w)^2(C_{L_\alpha})_{ss,t}$$

$$C_{L_\alpha} = (C_{L_\alpha})_{ss,w} + (C_{L_\alpha})_{ss,t}\left[1 - \left(\frac{\partial\epsilon}{\partial\alpha}\right)_{ss}\right]$$

$$C_{m_\alpha} = -(\ell/c_w)(C_{L_\alpha})_{ss,t}\left[1 - \left(\frac{\partial\epsilon}{\partial\alpha}\right)_{ss}\right] + (C_{m_\alpha})_f$$

Parameter Extraction

Equations (17) and (18) are to be used for parameter extraction. The procedure is to measure flight data as a function of time, convert the data to the frequency domain using the Fourier integral, and then use a maximum likelihood algorithm to estimate aerodynamic parameters that provide a fit to the data.[7-8]

In principle, it should be possible to extract the parameters $C_{Lq}, C_{L\alpha}, C_{m\alpha}, C_{mq}, C_{L\delta_e}, C_{m\delta_e}$, and the product $(C_{L\alpha})_{ss,t}(\partial\epsilon/\partial\alpha)_{ss}$ that appear in the equations of motion. The primary purpose of this study was to determine if including unsteady aerodynamics in parameter-extraction algorithms would result in different answers than would be obtained if unsteady aerodynamics were neglected. This was studied first by use of computer-generated data, and then by use of flight data.

Computer-Generated Data

Frequency-response characteristics $\tilde{\alpha}$ and $\tilde{q}$ for elevator input $\tilde{\delta}_e$ were computed for an aircraft having the geometry, inertias, and aerodynamic characteristics (unsteady downwash only) of Tables 1 and 2. The results are shown in Fig. 1. These data were then used with two parameter extraction algorithms: one including unsteady aerodynamics, the other neglecting unsteady aerodynamics.

In performing parameter estimation on these data, initial values of the parameters were offset from the values used to generate the data. Although the extraction program retrieved the values used in generating the data, several of the parameters were highly correlated. Pair-wise correlations above 0.95 were obtained between $C_{m\alpha}, C_{mq}$, and $(C_{m\alpha})_{ss,t} \times (\partial\epsilon/\partial\alpha)_{ss}$. The probability of high correlations between these parameters had been anticipated because of the manner in

Table 1 Geometric and mass characteristics of aircraft[a]

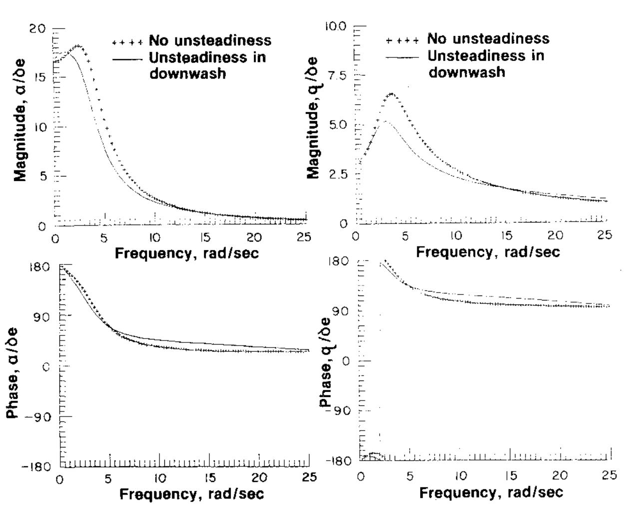
Characteristic	Wing	Horizontal tail
Aspect ratio	7.35	4.21
Taper ratio	1.00	1.00
Sweep angle, deg	0	0
Chord, m	1.34	0.77
Area, m^2	13.56	2.51

[a] Weight = 9230 N, $I_y = 2135$ kg m^2, $\ell = 4.4$ m.

Table 2 Aerodynamic characteristics and flight conditions[a]

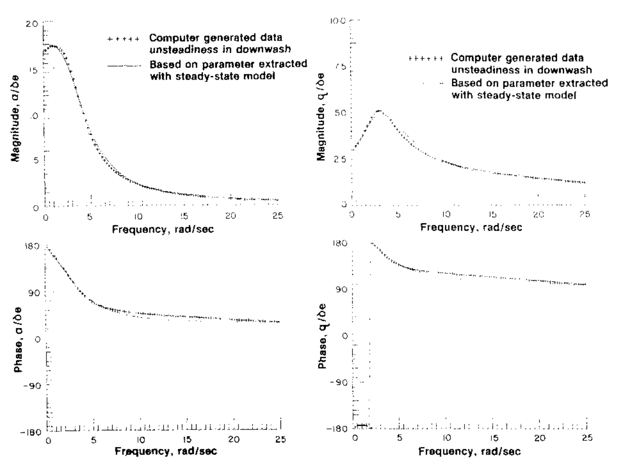
$(C_{L_\alpha})_{ss}$	=	5.21	C_{m_α}	=	-1.061
$(C_{L_\alpha})_{ss,t}$	=	0.74	C_{m_q}	=	-15.96
$C_{L_{\delta_e}}$	=	0.74	$C_{m_{\delta_e}}$	=	-2.43
C_{L_q}	=	-4.86	$(C_{m_\alpha})_f$	=	0.30
$\left(\dfrac{\partial\epsilon}{\partial\alpha}\right)_{ss}$	=	0.44	$(C_{L_\alpha})_{ss,t}\left(\dfrac{\partial\epsilon}{\partial\alpha}\right)_{ss}$	=	0.326

[a] $F = 1.4636$, $G = 0.530$, $H = 0.065$, $\rho = 1.076$ kg/m^3, and $U = 47.5$ m/s.



(a) Angle of Attack (b) Pitch Rate

Fig. 1 Frequency response curves.



(a) Angle of Attack (b) Pitch Rate

Fig. 2 Computer generated data in frequency domain.

which the terms appear in the equations. The extraction program was rerun with the product $(C_{m_\alpha})_{ss,t}(\partial\epsilon/\partial\alpha)_{ss}$ fixed at its correct value. This time the program again retrieved the correct aerodynamic parameters with all correlation coefficients low.

Table 3 Extracted parameters from simulated data[a]

Parameter	True value	Extracted value[b]
C_{L_α}	5.21	5.07 (0.042)
C_{L_q}	4.86	7.33 (0.11)
$C_{L_{\delta_e}}$	0.74	0.74 (0.03)
C_{m_α}	-1.061	-0.69 (0.01)
$C_{m_{\delta_e}}$	-2.43	-2.31 (0.02)
C_{m_q}	-15.96	-24.06 (0.20)

[a] Data generated with unsteadiness in downwash. [b] Extraction program neglected all unsteady effects.

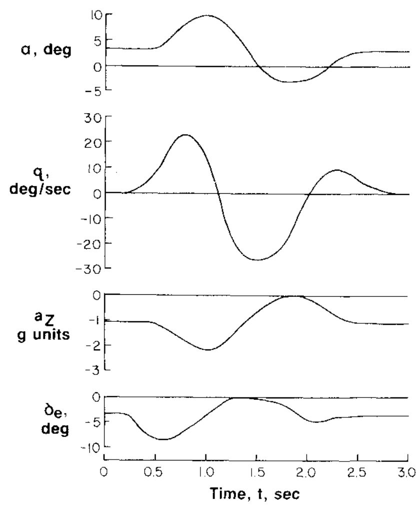


Fig. 3 Time histories of data measured from aircraft.

The next step in this study was to use the same computer-generated data (unsteadiness only in the downwash), and the parameter extraction that neglected unsteady effects ($\tilde{D} = 0$). It was found that the algorithm did extract a set of parameters that fit the data very closely, as shown in Fig. 2. However, as expected, some of the extracted parameters were different from those used in generating the data (Table 3). In particular, the damping-in-pitch parameter C_{m_q} is more negative than the value used in generating the data. This effect was anticipated since the extracted C_{m_q} must reflect the unsteady downwash effects. The manner in which the unsteady effects enter the pitching moment equation suggests that the additional damping can be interpreted as $C_{m_{\dot\alpha}}$. An approximate expression for $C_{m_{\dot\alpha}}$, generally referred to as the lag-in-downwash, is[9]

$$C_{m_{\dot\alpha}} = -2\left(\frac{\ell}{c_w}\right)^2\left(\frac{\partial\epsilon}{\partial\alpha}\right)_{ss}(C_{L_\alpha})_{ss,t} \tag{19}$$

If the parameters of Tables 1 and 2 are substituted into Eq. (19), the estimated value for $C_{m_{\dot\alpha}}$ is -7.02, which is reasonably close to the difference between the two extracted values of Table 3 (that is, a difference of -8.1). It should be noted that in this parameter-extraction exercise the geometric relationship $C_{m_q} = (-\ell/c_w)C_{L_q}$ was maintained.

The reduced value extracted for C_{m_α} when unsteady aerodynamics are not in the extraction algorithm was also of the expected order of magnitude. In this case the value should be

$$C_{m_\alpha} = (C_{m_\alpha})_{ss} - [(\rho S_w c_w C_{m_{\dot\alpha}} C_{L_\alpha})/4m] \tag{20}$$

Based on the characteristics of Tables 1 and 2, Eq. (20) results in $C_{m_\alpha} = -0.85$ which is reasonably close to the value of -0.69 actually extracted.

Table 4 Aerodynamic parameters extracted from flight data

Parameter	Extracted values	
	Case I[a]	Case II[b]
C_{L_α}	5.12 (0.05)	4.91 (0.03)
C_{L_q}	4.69 (0.17)	6.33 (0.33)
$C_{L_{\delta_e}}$	1.41 (0.07)	1.14 (0.05)
C_{m_α}	−1.34 (0.01)	−0 .99 (0.01)
$C_{m_{\delta_e}}$	−3.25 (0.05)	−2.95 (0.01)
C_{m_q}	−15.37 (0.31)	−20.77 (0.58)

[a]Extraction algorithm includes unsteadiness in downwash. [b]Extraction algorithm neglects unsteadiness in downwash.

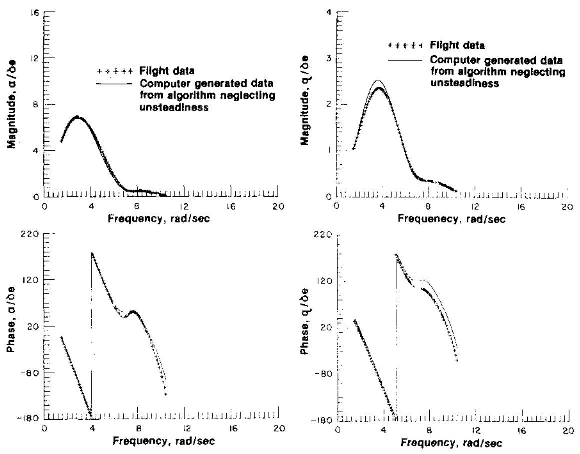


Fig. 4 Flight test data in frequency domain (unsteadiness neglected).

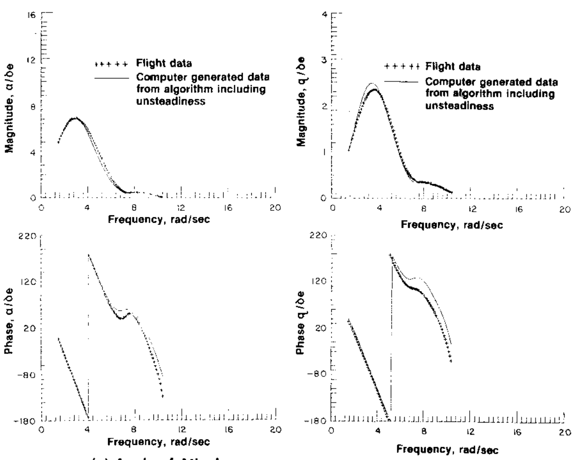


Fig. 5 Flight test data in frequency domain (unsteadiness included).

Flight Data

The flight data were obtained for a general-aviation light airplane. The geometric, mass, and flight conditions are given in Tables 1 and 2 and the measured flight data are shown in Fig. 3 as time histories of various measured states. The data of Fig. 3 were converted to the frequency domain by use of the Fourier integral, and are presented in Figs. 4 and 5.

Parameter extraction was performed in two different modes: 1) in the frequency domain, using the algorithm with unsteady aerodynamic terms included and 2) in the frequency domain, using the algorithm with unsteady aerodynamic terms omitted.

First, an attempt was made to extract all six parameters of Eqs. (17) and (18) plus the product $(C_{L_\alpha})_{ss,t}(\partial\epsilon/\partial\alpha)_{ss}$. The problem of high correlations was encountered, as in the case with computer-generated data and some of the extracted parameters were unrealistic in magnitude and sign.

Parameter extraction was then performed with the product $(C_{L_\alpha})_{ss,t}(\partial\epsilon/\partial\alpha)_{ss}$ held constant at a value estimated based on the wing and tail geometry. The parameters extracted in the two modes are given in Table 4, and the frequency-response characteristics obtained are shown in Figs. 4 and 5. Although some of the parameters are appreciably different, both sets fit the flight-measured time histories equally well. The trends shown in the parameters in going from including-to-omitting unsteady aerodynamics are very similar to those observed from the results from generated data (Table 3). The most pronounced effect was again in the increased (more negative) value of C_{m_q} obtained when unsteady aerodynamics are omitted from the extraction algorithm.

Concluding Remarks

A simple vortex system has been used to model unsteady incompressible aerodynamic effects into the longitudinal equations of motion of an aircraft. The equations of motion in the time domain had two difficulties relative to application for parameter extraction: 1) a limited number of aerodynamic parameters appeared in the equations; and 2) the equations were integro-differential equations which would lead to computational difficulties. The second problem was circumvented by transforming to the frequency domain; however, the first problem (limited number of derivatives) remained. Subsequent calculations showed that unsteady aerodynamic effects were adequately modeled if the unsteady effects were included in the downwash modeling. This permitted recasting the equations so that the usual aerodynamic parameters were present.

Computer-generated data, and real flight data were used to demonstrate that inclusion of unsteady aerodynamics in the parameter-extraction algorithm would produce aerodynamic parameters that were different from those extracted if unsteady aerodynamics were left out of the algorithm. Modeling unsteady aerodynamics into the equations of motion permitted extraction of aerodynamic derivatives associated with steady-state motion.

The differences between derivatives associated with the two extraction algorithms (with and without unsteady aerodynamics) can be related to the acceleration derivatives $C_{L_{\dot\alpha}}$ and $C_{m_{\dot\alpha}}$. It was also shown that the difference in C_{m_q}, obtained from the two algorithms, could be approximated closely by use of the lag-in-downwash concept.

References

[1]Morino, L., "A General Theory of Unsteady Compressible Potential Aerodynamics," NASA CR-2469, 1974.

[2]Piezkin, S.T. and Levinsky, E.S., "Nonlinear Lifting Line Theory for Predicting Stalling Instabilities on Wings of Moderate Aspect Ratio," General Dynamics Rept. CASD-NSC-76-001, June 1976.

[3]Edwards, J. W., "Unsteady Aerodynamic Modeling and Active Aeroelastic Control," SUDAR 504, Feb. 1977.

[4]Wells, W.R., and Queijo, M.J., "Simplified Unsteady Aerodynamic Concepts, With Application to Parameter Estimation," *Journal of Aircraft*, Vol. 16, Feb. 1979, pp. 90-94.

[5]Queijo, M.J., Wells, W.R., and Keskar, D.A., "Approximate Indicial Life Function for Tapered, Swept Wings in Incompressible Flow," NASA TP-1241, 1978.

[6]Statler, I.C., "Dynamic Stability at High Speeds From Unsteady Flow Theory," *Journal of the Aeronautical Sciences*, April 1950.

[7]Keskar, D.A. and Wells, W.R., "Maximum Likelihood Identification of Aircraft Parameters With Unsteady Aerodynamic Modelling," AIAA Paper 79-0400, AIAA 17th Aerospace Science Meeting, New Orleans, La., Jan. 1979.

[8]Klein, V., "Aircraft Parameter Estimation in the Frequency Domain," AIAA Paper 78-1344, AIAA Fifth Atmospheric Flight Mechanics Conference, Palo Alto, Calif., Aug. 1978.

[9]Etkin, B., *Dynamics of Atmospheric Flight*, John Wiley and Sons, Inc., 1972, pp. 283-284.